**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans :

X=50, Mean=45, Std=8

from scipy import stats

stats.norm.cdf(50,loc=45,scale=8)

0.7340144709512995

1-0.7340144709512995 = 0.26598552904870054

= 0.2659(B)

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Ans:

from scipy import stats

400\*(1-stats.norm.cdf(44,loc=38,scale=6))

63.46210157258283

400\*(stats.norm.cdf(44,loc=38,scale=6)-stats.norm.cdf(38,loc=38,scale=6))

136.53789842741716

(63.46<136.53) **False**

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: **True**

from scipy import stats

400\*(stats.norm.cdf(30,loc=38,scale=6))

36.484487890347154

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans:

* If X1 is normally distributed then 2X1 is also normally distributed and 2X1 is larger scale of X1.
* X1 and X2 are normally distributed and their associated sum and samples are exactly normal with suitable parameters.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans:

Range for 99% of data lies between 3rd std of Mean

Mean=100, std=20

(100**±**3\*20) = (100-60, 100+60)

= (40,160) (D)

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans:

import numpy as np

from scipy.stats import norm

from scipy import stats

mean = (5+7) \* 45

print(mean)  
 540

sd = ((3^2)+(4^2)) \* 45

print(sd)

315

a, b = norm.interval(alpha=0.95, loc=540, scale=315)

print(a,b)

-77.38865513011706 1157.388655130117

Range = (-77.38, 1157.388) Million Rupees.

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans:

To compute the 5th percentile, we use the formula X= µ+Zσ, from Ztable 5th percentile= -1.64

X = (-1.645\*315)+540

print(X)

21.825000000000045

=21.82 million rupees.

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans:

stats.norm.cdf(0,5,3)

0.0477903522728147

stats.norm.cdf(0,7,4)

0.040059156863817086

Division 1 making a loss in a given year is more than Division 2.